Sikkim Public Service Commission

Main Written Examination for the Post of Commercial Tax Inspector

Mathematics

Paper - II

Time Allowed : 3 Hrs. | Maximum Marks : 300

INSTRUCTIONS TO CANDIDATES

Read the following instructions carefully before answering the questions :-

1. IMMEDIATELY AFTER THE COMMENCEMENT OF THE EXAMINATION, YOU SHOULD CHECK THAT THIS TEST BOOKLET DOES NOT HAVE ANY UNPRINTED OR TORN OR MISSING PAGES OR ITEMS ETC. IF SO, GET IT REPLACED BY A COMPLETE TEST BOOKLET.

2. Please note that it is the candidate's responsibility to fill in the Roll Number and Test Booklet Serial Number carefully and without any omission or discrepancy at the appropriate places in the **OMR ANSWER SHEET.**

3. **Use only Black Ball Point Pen to fill the OMR sheet**

4. Do not write anything else on the OMR Answer Sheet except the required information.

5. **This Test Booklet contains 3 Sections. Section A is of Multiple choice Question i.e. 100 items to be marked in OMR Sheet. Section B is Short Answer type Questions. Section C is Long Answer/ Essay type Questions, which has to be written in Seperate Answer Sheet provided.**

6. **All items from Q.1 to Q. 100 carries 2 marks each.**

7. Before you proceed to mark in the Answer Sheet (OMR), you have to fill in some particulars in the Answer Sheet (OMR) as per given instructions.

8. After you have completed filling in all your responses on the Answer Sheet (OMR) and the examination has concluded, you should hand over the Answer Sheet (OMR) and the Seperate conventional Answer sheet to the Invigilator only . You are permitted to take away with you the Test Booklet.

9. **Marking Scheme**

   There will be negative marking for wrong answers marked by a candidate in the objective type question papers.

   (i) There are four alternatives for the answer to every question. For each question for which a wrong answer has been given by the candidate, one-third of the marks assigned to that question will be deducted as penalty.

   (ii) If a candidate gives more than one answer, it will be treated as a wrong answer even if one of the given answers happens to be correct and there will be same penalty as above to the question.

   (iii) If a question is left blank. i.e., no answer is given by the candidate, there will be no penalty for that question.

DO NOT OPEN THIS TEST BOOKLET UNTIL YOU ARE ASKED TO DO SO

Section A (Multiple Choice Question)

1 Consider the function $f(x,y) = 5 - 4sinx + y^2$ for $0<x<2$ and $y \in \mathbb{R}$. The set of critical points of $f(x,y)$ consists of

(A) a point of local maximum and a point of local minimum

(B) a point of local maximum and a saddle point

(C) a point of local maximum , a point of local minimum and a saddle point

(D) a point of local minimum and a saddle point

2. Let $\varphi:\mathbb{R}\to\mathbb{R}$ be a differentiable function such that φ is strictly increasing with $\varphi'(1) = 0$.

Let and β denote the minimum and maximum values of $\varphi(x)$ on the interval [2, 3], respectively. Then which of the following is TRUE?

(A) $\beta = (3)$ (B) $\alpha = \varphi(2.5)$

(C) $\beta = (2.5)$ (D) $\alpha = \varphi(3)$

3. $\lim_{n\to\infty} \frac{\pi}{n} \sum_{k=1}^{n} \sin(\frac{\pi}{2} + \frac{5\pi}{2} . \frac{k}{n})$ is equal to

(A) $\frac{2\pi}{5}$ (B) $\frac{5}{2}$

(C) $\frac{2}{5}$ (D) $\frac{5\pi}{2}$

4. Let $f_1(x), f_2(x), g_1(x)$ *and* $g_2(x)$ be differentiable functions on $\mathbb{R}$. Let F(x) =

$\begin{vmatrix} f_1(x) & f_2(x) \\ g_1(x) & g_2(x) \end{vmatrix}$ be the determinant of the matrix $\begin{bmatrix} f_1(x) & f_2(x) \\ g_1(x) & g_2(x) \end{bmatrix}$ Then $F'(x)$ is equal to

(A) $\begin{vmatrix} f_1'(x) & f_2'(x) \\ g_1(x) & g_2(x) \end{vmatrix} + \begin{vmatrix} f_1(x) & g_1'(x) \\ f_2'(x) & g_2(x) \end{vmatrix}$

(B) $\begin{vmatrix} f_1'(x) & f_2'(x) \\ g_1(x) & g_2(x) \end{vmatrix} + \begin{vmatrix} f_1(x) & g_1'(x) \\ f_2(x) & g_2'(x) \end{vmatrix}$

(C) $\begin{vmatrix} f_1'(x) & f_2'(x) \\ g_1(x) & g_2(x) \end{vmatrix} - \begin{vmatrix} f_1(x) & g_1'(x) \\ f_2(x) & g_2'(x) \end{vmatrix}$

(D) $\begin{vmatrix} f_1'(x) & f_2'(x) \\ g_1'(x) & g_2'(x) \end{vmatrix}$

5. If $f(x) = \begin{cases} 1 + x, & if\ x < 0 \\ (1-x)(px+q), & if\ x \geq 0 \end{cases}$ satisfies the assumptions of Rolle's Theorem in the interval [-1,1], then the ordered pair (p, q) is

(A) (2,-1) (B) (-2,-1)

(C) (-2, 1) (D) (2, 1)

6. The interval of convergence of the power series $\sum_{n=1}^{\infty} \frac{1}{(-3)^{n+2}} \frac{(4x-12)^n}{n^2+1}$ is

(A) $\frac{10}{4} \leq x < \frac{14}{4}$ (B) $\frac{9}{4} \leq x < \frac{15}{4}$

(C) $\frac{10}{4} \leq x \leq \frac{14}{4}$ (D) $\frac{9}{4} \leq x \leq \frac{15}{4}$

7. Let $f(x,y) = \frac{x^2}{x^2+y^2}$ $for(x,y) \neq (0,0)$,Then

(A) $\frac{\partial f}{\partial x}$ and f are bounded (B) $\frac{\partial f}{\partial x}$ is bounded and f is unbounded

(C) $\frac{\partial f}{\partial x}$ is unbounded and f is bounded (D) $\frac{\partial f}{\partial x}$ and f are unbounded

8. Let S be an infinite subset of $\mathbb{R}$ such that $S \setminus \{\alpha\}$ is compact for some $\alpha \in S$. Then which of the following is TRUE?

(A) S is a connected set

(B) S contains no limit points

(C) S is a union of open intervals

(D) Every sequence in S has a subsequence converging to an element in S.

9. Let $0 < a_1 < b_1$ $for\ n \geq 1, define$ $a_{n+1} = \sqrt{a_n b_n}$ and $b_{n+1} = \frac{a_n+b_n}{2}$ Then which of the following is NOT TRUE?

(A) Both $\{a_n\}$ and $\{b_n\}$ converge, but the limits are not equal.

(B) Both $\{a_n\}$ and $\{b_n\}$ converge, but the limits are equal.

(C) $\{b_n\}$ is a decreasing sequence

(D) $\{a_n\}$ is an increasing sequence

10. A particular integral of the differential equation $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} = e^{2x} sinx$ is

(A) $\frac{e^{2x}}{10}(3cosx - 2sinx)$

(B) $\frac{-e^{2x}}{10}(3cosx - 2sinx)$

(C) $\frac{-e^{2x}}{5}(2cosx + sinx)$

(D) $\frac{e^{2x}}{5}(2cosx - sinx)$

11 . Let y(x) be the solution of differential equation $(xy + y + e^{-x})dx + (x + e^{-x})dy = 0$ satisfying $y(0) = 1$. Then $y(-1)$ is equal to

(A) $\frac{e}{e-1}$ (B) $\frac{2e}{e-1}$

(C) $\frac{e}{1-e}$ (D) 0

12. Let $f:\mathbb{R}^2 \to \mathbb{R}$ be a function. Then which of the following statement is TRUE?

(A) If f is differentiable at (0,0), then all directional derivatives of f exist at (0,0)

(B) If all directional derivatives of f exist at (0,0) then f is differentiable at (0,0)

(C) If all directional derivatives of f exist at (0,0) then f is continuous at (0,0)

(D) If all the partial derivatives $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ exist and are continuous in a disc centered at (0, 0), then f is not differentiable at (0,0).

13. If X and Y are n x n matrices with real entries, then which of the following is FALSE?

(A) If $P^{-1}XP$ is diagonal for some real invertible matrix P, then there exist a basis for $\mathbb{R}$ V consisting of eigenvector of X.

(B) If X is diagonal with distinct diagonal entries and XY=YX, then Y is also diagonal

(C) If X^2 is diagonal then X is diagonal

(D) If X is diagonal and XY=YX for all Y, then $X=\lambda I$ for some $\lambda \in \mathbb{R}$

14. A random sample of size n is chosen from a population with probability density function

$$f(x,\theta) = \begin{cases} \frac{1}{2}e^{-(x-\theta)}, & x \geq \theta \\ \frac{1}{2}e^{(x-\theta)}, & x < \theta \end{cases}$$

Then the maximum likelihood estimation of θ is the

(A) Mean of the sample (B) Standard deviation of the sample

(C) Median of the sample (D) Maximum of the sample

15. Let $\{X_n\}$ be a sequence of independent random variables with

$$p(X_n = n^\alpha) = p(X_n = -n^\alpha) = \frac{1}{2}$$

The sequence $\{X_n\}$ obeys the weak law of large number if

(A) $\alpha < \frac{1}{2}$ (B) $\alpha = \frac{1}{2}$ (C) $\frac{1}{2} < \alpha \leq 1$ (D) $\alpha < 1$

16. Let $A_{1,2}, A_3 \ldots, A_n$ be n independent events which the probability of occurrence of the event A_i given by $P(A_i) = 1 - \frac{1}{\alpha^i}, \alpha > 1,\ i = 1,2,\ldots,n$. Then the probability that at least one of the events occurs is

(A) $1 - \frac{1}{\alpha^{\frac{n(n+1)}{2}}}$ (B) $\frac{1}{\alpha^{\frac{n(n+1)}{2}}}$ (C) $\frac{1}{\alpha^n}$ (D) $1 - \frac{1}{\alpha^n}$

17. Let $P(X = n) = \frac{\lambda}{n^2(n+1)}$, where λ an appropriate constant is. Then E(X) is

(A) $2\lambda+1$ (B) λ (C) ∞ (D) 2λ

18. There are two identical locks, with two identical keys, and the keys are among the six different ones which a person carries in his pocket. In a hurry he drops one key somewhere then the probability that the locks can still be opened by drawing one key at random is equal to

(A) $\frac{1}{5}$ (B) $\frac{5}{6}$ (C) $\frac{1}{30}$ (D) $\frac{1}{12}$

19. The random variable X has at t-distribution with υ degrees of freedom. Then the probability distribution of X^2 is

(A) Chi-square distribution with 1 degree of freedom
(B) Chi-square distribution with υ degrees of freedom
(C) F-distribution with $(1,\upsilon)$ degree of freedom
(D) F-distribution with $(\upsilon,1)$ degree of freedom

20. The possible set of eigen values of 4x4 skew symmetric orthogonal real matrix is

(A)$\{\pm i\}$ (B) $\{\pm i,\pm 1\}$ (C) $\{\pm 1\}$ (D) $\{0,\pm i\}$

21. The coefficient of $(z-\pi)^2$ in the Taylor series expansion of

$$f(z)=\begin{cases}\frac{\sin z}{z-\pi}, & if z\neq\pi\\ -1, & if z=\pi\end{cases}$$

around π is

(A) $\frac{1}{2}$ (B) $\frac{-1}{2}$ (C) $\frac{1}{6}$ (D) $\frac{-1}{6}$

22. Consider the linear programming problem:

Maximize $x+\frac{3}{2}y$
Subject to

$$2x+3y\leq 16,$$
$$x+4y\leq 18,$$
$$x\geq 0,\geq 0$$

If S denotes the set of all solutions of the above problems, then

(A) S is empty
(B) S is a singleton
(C) S is a line segment
(D) S has positive area

23. The probability that A happens is 1/3. The odds against happening of A are

(A) 2:1 (B) 2:3

(C) 3:2 (D) 5:2

24. The probability that A passes a test is 2/3 and the probability that B passes the same test is 3/5. The probability that only one of them passes is

(A) 2/5 (B) 4/15

(C) 2/15 (D) 7/15

25. The general solution to the ordinary differential equation $x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}+\left(4x^2-\frac{9}{25}\right)y=0$

in terms of Bessel's functions $J_n(x)$ is

(A) $y(x)=C_1J_{3/5}(2x)+C_2J_{-3/5}(2x)$

(B) $y(x)=C_1J_{3/10}(x)+C_2J_{-3/10}(x)$

(C) $y(x)=C_1J_{3/5}(x)+C_2J_{-3/5}(x)$

(D) $y(x)=C_1J_{3/10}(2x)+C_2J_{-3/10}(2x)$

26. Let M be an invertible Hermitian matrix and let $x, \in \mathbb{R}$ be such that $x^2<4y$. Then

(A) Both $M^2+xM+yI$ and $M^2-xM+yI$ are singular

(B) $M^2+xM+yI$ singular but $M^2-xM+yI$ is non-singular

(C) $M^2+xM+yI$ non-singular but $M^2-xM+yI$ is singular

(D) Both $M^2+xM+yI$ and $M^2-xM+yI$ are non-singular

27. Let M be a 3x3 matrix and suppose that 1, 2 and 3 are the eigen values of M. If

$M^{-1}=\frac{M^2}{\alpha}-M+\frac{11}{\alpha}I_3$ for some scalar $\alpha\neq0$, then α is equal to

(A) 9 (B) 4 (C) 6 (D) 8

28. The objective function of a linear programming model is given as:

Maximize $z = x_1 + 2x_2$
Subject to

$$x_1 + x_2 \leq 5,$$
$$x_1 + 3x_2 \leq 9$$

What is the objective function value if $(x_1, x_2) = (1, 1)$ is used as a possible solution.

(A) 3 (B) 5 (C) 7 (D) None of the above

29. Following is an infeasible solution for the linear programming model given below:

Maximize $z = x_1 + 2x_2$
Subject to

$$x_1 + x_2 \leq 5,$$
$$x_1 + 3x_2 \leq 9$$

(A) $(x_1, x_2) = (1, 3)$

(B) $(x_1, x_2) = (3, 1)$

(C) $(x_1, x_2) = (1, 1)$

(D) None of the above

30. Which if the following statement is not true for a graphical solution to a linear programming model:

(A) In a minimization problem, the optimal solution improves if the objective function line is brought closer to the origin

(B) A graphical solution method can solve any linear programming problem without any restrictions on the number of decision variables involved

(C) In a maximization problem, the value of objective function improves as we move away from origin.

(D) None of the above

31. A linear programming model given below:
Maximize $z = x_1 + 2x_2$
Subject to

$$x_1 + x_2 \leq 5,$$
$$x_1 + 3\,x_2 \leq 9$$

What is the objective function value if $(x_1, x_2) = (3, 1)$ is used as a possible solution?

(A) 4 (B) 5 (C) 7 (D) None of the above

32. In a problem with 2 decision variables, the 100% rule indicates that each coefficient can be safely increased by __________________ without invalidating the current optimal solution?

(A) 50%

(B) 50% of the allowable increase of that coefficient

(C) 100%

(D) 50% of the range of optimality

33. Resource-allocation problems typically have which of the following type of constraints

(A) >= (Benefit Constraint) (B) <= (Resource Constraint)

(C) = (Fixed Requirement Constraint) (D) All of the above

34. Given the following two constraints, which solution is a feasible solution for a maximization problem?
Constraint 1: $4\ x_1 + 3_2 \leq 18$,
Constraint 2: $x_1 - x_2 \leq 3$
(A) $(x_1, x_2) = (1, 5)$.
(B) $(x_1, x_2) = (4, 1)$.
(C) $(x_1, x_2) = (4, 0)$.
(D) $(x_1, x_2) = (2, 1)$.

35. Let A and B be two events such that P(A) = $\frac{1}{5}$ While P(A or B) = $\frac{1}{2}$. Let P(B) = P. For what values of P are A and B independent?
(A) $\frac{1}{10}$ and $\frac{3}{10}$
(B) $\frac{3}{10}$ and $\frac{4}{5}$
(C) $\frac{3}{8}$ only
(D) $\frac{3}{10}$

36. A box has 6 black, 4 red, 2 white and 3 blue shirts. What is the probability that 2 red shirts and 1 blue shirt get chosen during a random selection of 3 shirts from the box?
(A) 18/455
(B) 7/15
(C) 7/435
(D) 7/2730

37. Tickets numbered 1 to 50 are mixed and one ticket is drawn at random. Find the probability that the ticket drawn has a number which is a multiple of 4 or 7?
(A) 9/25
(B) 9/50
(C) 18/25
(D) None of these

38. What is the possibility of having 53 Thursdays in a non-leap year?
(A) 6/
(B) 1/7
(C) 1/365
(D) 53/365

39. If $f{:}R \to R$ is such that $f(0) = 0$ and $\left|\frac{d}{dx}f(x)\right| \leq 5\ \forall x$.
We can conclude that (1) is in
(A) (5, 6) (B) [-5, 5]
(C) $(-\infty,-5) \cup (5, \infty)$ (D) [-4, 4]

40. If A=$\{(x,y): y = e^x, x \in R\}$ and B=$\{(x,y): y = e^{-x}, x \in R\}$ then

(A) $A \cap B = \emptyset$

(B) $A \cap B \neq \emptyset$

(C) $A \cup B = R \times R$

(D) None of these

41. The limit superior and limit inferior of $\langle\frac{(-1)^n}{n^2}\rangle$ are respectively equal to

(A) -1, -1
(B) !, 0
(C) 0, 0
(D) 1, 1

42. The sequence $\langle S_n\rangle$, where $S_n = 1 + \frac{1}{3} + \frac{1}{5} + \ldots + \frac{1}{2n-1}$ is

(A) Convergent

(B) Monotonic increasing

(C) Not Cauchy

(D) None of these

43. For which real number t does the infinite series $\sum \frac{\overline{n+1} - \bar{n}}{n^t}$

(A) t > 1/3

(B) t > 1/2

(C) t > 1

(D) t > 3/2

44. Let S be an uncountable set and T be a set of those real number x such that $(x-\delta, +\delta) \cap S$ is uncountable then which of the statement is/are correct

i. T is countable

ii. S-T is countable

iii. $S \cap T$ is countable

(A) Only (i) and (ii)

(B) Only (ii) and (ii)

(C) Only (i) and (iii)

(D) All are correct

45. If the sequence $\langle a_n \rangle$ be defined by $a_1 = 1$ and $a_{n+1} = \sqrt{a_n + 6}$ for n>1, then $\lim_{n \to \infty} a_n$ is equal to

(A) 1
(B) 2
(C) 3
(D) 4

46. $\lim_{x \to 0} \frac{(1+x)^{1/2} - e + \frac{ex}{2}}{x^2}$ is

(A) $\frac{24}{11}e$
(B) $\frac{11}{24}e$
(C) $\frac{1}{11}e$
(D) $\frac{1}{24}e$

47. The radius of convergence of the series $\sum_{n=1}^{\infty} Z^{n^2}$ is

(A) 0
(B) ∞
(C) 1
(D) 2

48. The matrix $\begin{bmatrix} 3 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 3 \end{bmatrix}$ is

(A) Positive definite
(B) Non-negative definite but not positive definite
(C) Negative definite
(D) Neither negative definite nor positive definite

49. Let $\Delta_1 = \begin{vmatrix} a & b & c \\ x & y & z \\ p & q & r \end{vmatrix}$, $\Delta_2 = \begin{vmatrix} -x & a & -p \\ y & -b & q \\ z & -c & r \end{vmatrix}$. Then

(A) $\Delta_1 = \Delta_2$
(B) $\Delta_1 = 2\Delta_2$
(C) $\Delta_1 = -\Delta_2$
(D) $2\Delta_1 = \Delta_2$

50. Let A be a real 3×4 matrix of rank 2. Then the rank of $A^T A$, where A^T denotes the transpose of A is

(A) Exactly 2
(B) Exactly 3
(C) At most 2 but not necessary 2
(D) Exactly 4

51. If A is a real 5×5 matrix with trace 15 and if 2 and 3 are eigen values of A, each with algebraic multiplicity 2, then the determinant of A is

(A) 0
(B) 24
(C) 120
(D) 180

52. A matrix has eigen values 1 and 4 with corresponding eigenvectors $(1, -1)^T$ and $(2, 1)^T$, respectively. Then M is

(A) $\begin{bmatrix} -4 & -8 \\ 5 & 9 \end{bmatrix}$ (B) $\begin{bmatrix} 9 & -8 \\ 5 & -4 \end{bmatrix}$

(C) $\begin{bmatrix} 2 & 2 \\ 1 & 3 \end{bmatrix}$ (D) $\begin{bmatrix} 3 & 2 \\ 1 & 2 \end{bmatrix}$

53 Let = $\begin{bmatrix} 1 & 1+i & 2i \\ 1-i & 3 & 4 \\ 9 & 7+i & -i \end{bmatrix}$. Then

(A) M has purely imaginary eigen values

(B) M is not diagonalizable

(C) M has eigen values which are neither real nor purely imaginary

(D) M has only real eigen values.

54. The matrix of the quadratic form $ax^2 + by^2 + cz^2 + 2hxy + 2gx + 2fy + c$ is

(A) $\begin{bmatrix} a & g & h \\ h & b & f \\ g & f & c \end{bmatrix}$ (B) $\begin{bmatrix} a & h & g \\ b & h & f \\ g & f & c \end{bmatrix}$

(C) $\begin{bmatrix} a & h & g \\ h & b & f \\ g & f & c \end{bmatrix}$ (D) $\begin{bmatrix} a & h & g \\ h & b & f \\ f & g & c \end{bmatrix}$

55. The system of equations

$$x + y + z = 0, 3x + 6y + z = 0, \alpha x + 2y + z = 0$$

has infinitely many solutions the α is equal to

(A) 7 (B) $\frac{7}{5}$

(C) $\frac{5}{7}$ (D) 4

56. An integrating factor for $ydx - xdy = 0$ is

(A) $\frac{x}{y}$ (B) $\frac{y}{x}$

(C) $\frac{1}{x^2+y^2}$ (D) $\frac{1}{x^2+y}$

57. The value of $\iint_R dxdy$ where R is the rectangular region bounded between lines x = 0, x = 2, y = 0, y = 3 is

(A) 5
(B) 9
(C) 4
(D) 6

58. If $\vec{r} = xi + yj + zk$ and $r = |\vec{r}|$, then divergence of $\frac{\vec{r}}{|\vec{r}|}$ is

(A) 5
(B) 9
(C) 0
(D) 6

59. If $\vec{f} = x^2y\hat{\imath} - 2xz\hat{\jmath} + 2yz\hat{k}$, then $\nabla \times (\nabla \times \vec{f})$ is

(A) (2x+2) $\hat{\imath}$
(B) (2x+2)$\hat{\jmath}$
(C) (2x+2) $\hat{k}$
(D) None of these

60. Divergence theorem correlates:

(A) line integral and double integral

(B) line integral and surface integral

(C) volume integral and surface integral

(D) double integral and surface integral

61. Equation of the plane passing through the point (3, -3, 1) and parallel to the plane $2x + 3y + 5z + 6 = 0$ is

(A) $2x + 3y + 5z + 2 = 0$
(B) $2x + 3y + 5z - 2 = 0$
(C) $2x + 3y + 5z + 3 = 0$
(D) $2x + 3y + 5z + 4 = 0$

62. Let A be the matrix of quadratic form $(x - y + 2z)^2$. Then trace of A is

(A) 2
(B) 4
(C) 6
(D) 0

63. Let $T: R^7 \rightarrow R^7$ be a linear transformation such that $T^2 = 0$. Then the rank of T is

(A) ≤ 3
(B) > 3
(C) =5
(D) = 6

64. Laplace transform of $e^{2t}cos^2 t$ is

(A) $\frac{1}{2}\{\frac{1}{s-2}+\frac{s-2}{(s-2)^2+4}\}$

(B) $\frac{1}{2}\{\frac{1}{s+2}+\frac{s+2}{(s+2)^2+4}\}$

(C) $\frac{1}{2}\{\frac{1}{s}+\frac{s}{(s-2)^2+4}\}$

(D) $\frac{1}{2}\{\frac{1}{s}+\frac{s-2}{s^2+4}\}$

65. If $f(x)$ can be expressible in the fourier series then, which of the following is not true

(A) f(x) is periodic function

(B) f(x) has finite many discontinuities

(C) f(x) is single valued function

(D) f(x) is multiple valued function

66. If $f(x) = e^{-x}$ can be expressible in the interval (0, π), then the coefficients b_n is equal to

(A) $\frac{n}{n^2+1}$

(B) $\frac{1-e^{-2\pi}}{\pi}$

(C) $\frac{n}{n^2+1}(\frac{1-e^{-2\pi}}{\pi})$

(D) None of these

67. If $\lambda_1, \lambda_2, ..., \lambda_n$ are eigenvalues of matrix A then determinant of A is

(A) $\lambda_1\lambda_2 ... \lambda_n$

(B) $\lambda_1 + \lambda_2 + ... + \lambda_n$

(C) $\sqrt{\lambda_1 + \lambda_2 + ... + \lambda_n}$

(D) $\sqrt{\lambda_1\lambda_2 ... \lambda_n}$

68. If A and B are Hermitian matrices then which of the following is not true

(A) AB+BA is Hermitian

(B) AB-BA is Skew Hermitian

(C)$B^\theta B$ is Hermitian

(D) A+A^θ is Skew Hermitian

69. If $X = AY$. Where $A = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$; $x = (x_1, x_2)^T$ and $y = (y_1, y_2)^T$; then $x_1^2 + x_2^2$ transform to

(A) $\sqrt{2}(y_1^2 + y_2^2)$

(B) $(y_1 - y_2)^2$

(C) $y_1^2 + y_2^2 + y_1y_2$

(D) $y_1^2 + y_2^2$

70. The complete integral of $(p+q)(z-xp-yq)=1$ is

(A) $z = ax + by + a + b$

(B) $z + ax - by = \frac{1}{a+b}$

(C) $z = ax - by + \frac{1}{a+b}$

(D) $z = ax + by + \frac{1}{a+b}$

71. The general solution of $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ is

(A) $u = f(x+iy) + g(x-iy)$

(B) $u = f(x+y) + g(x-y)$

(C) $z = cf(x-iy)$

(D) $u = g(x+y)$

72. The maximum value of $P = 4x + 2y$ subject to $4x + 2y \le 46, x + 3y \le 24, x \ge 0,$ $y \ge 0$ occur at

(A) One point

(B) Two point

(C) Four point

(D) Infinite number of point

73. In a LPP, if the objective function is parallel to a constraint and the feasible region is non empty, then the objective function may assume

(A) An unbounded solution

(B) Unique optimal solution

(C) Multiple optimal solution

(D) Any one of the above

74. The function $f(x) = C^T x + d$ is

(A) Convex

(B) Concave

(C) Both convex and concave

(D) Either convex and concave

75. The third central moment of the binomial distribution $B(1, \frac{1}{3})$ is

(A) $\frac{1}{27}$

(B) $\frac{1}{3}$

(C) $\frac{2}{27}$

(D) $\frac{5}{27}$

76. If X is a poisson random variate with mean 3, then $P(|X-3|<1)$ will be

(A) $\frac{1}{2}e^{-3}$

(B) $3e^{-3}$

(C) $\frac{9}{2}e^{-3}$

(D) $\frac{99}{8}e^{-3}$

77. Let X be a normal random variable with mean zero and variance 9. If $a = P(X \geq 3)$, then

$P(|X| \leq 3)$ equals

(A) a
(B) 1-a
(C) 2a
(D) 1-2a

78. If A and B are independent events, $P(A) = 0.5$ and $P(A \cup B) = 0.6$, then $P(B)$ is

(A) 0.1
(B) 0.2
(C) 0.3
(D) 0.5

79. If $f = tan^{-1}(\frac{y}{x})$ then div (grad f) is equal to

(A) 1
(B) -1
(C) 0
(D) 2

80. A necessary and sufficient condition that the line integral $\int_C \vec{F}.\overrightarrow{dR}$ along C vanishes is

(A) curl$\vec{F} = 0$
(B) div$\vec{F} = 0$
(C) curl$\vec{F} \neq 0$
(D) div$\vec{F} \neq 0$

81. The work done by the force $\vec{F} = yz\hat{\imath} + zx\hat{\jmath} + xy\hat{k}$ in moving a particle from the point (1, 1, 1) to the point (3, 3, 2) along the path C is

(A) 17
(B) 10
(C) 0
(D) cannot be defined

82. Value of $\int_C y^2dx + x^2dy$, where c is the boundary of the square $-1 \leq x, y \leq 1$ is

(A) 4
(B) 0
(C) 2(x+y)
(D) 4/3

83. The magnitude of the vector drawn perpendicular to the surface $x^2 + 2y^2 + z^2 = 7$ at the point (1, -1, 2) is

(A) 2/3
(B) 3/2
(C) 3
(D) 6

84. Value of $\Gamma\left(\frac{1}{4}\right).\Gamma\left(\frac{3}{4}\right)$ is equal to

(A) $\sqrt{\pi/2}$

(B) $\sqrt{2/\pi}$

(C) $\pi\sqrt{2}$

(D) None of these

85. Value of $\beta\left(\frac{5}{2},\frac{7}{2}\right)$ is equal to

(A) $\frac{3\pi}{256}$

(B) $\frac{2\pi}{256}$

(C) $\frac{\pi}{256}$

(D) None of these

86. A unit vector perpendicular to the vectors $-2\hat{\imath}+3\hat{\jmath}+\hat{k}$ and $4\hat{\imath}+2\hat{\jmath}$ is

(A) $\frac{-2\hat{\imath}+3\hat{\jmath}+\hat{k}}{\sqrt{69}}$

(B) $\frac{\hat{\imath}+3\hat{\jmath}+\hat{k}}{\sqrt{96}}$

(C) $\frac{\hat{\imath}-2\hat{\jmath}-8\hat{k}}{\sqrt{69}}$

(D) None of these

87. The value of $\lim_{x\to\pi/2}\frac{\log sinx}{(\frac{\pi}{2}-x)^2}$ is

(A) 0

(B) 1/2

(C)-1/2

(D) -2

88. Taylor's expansion of the function $f(x)=\frac{1}{x^2+1}$ is

(A) $\sum_{n=0}^{\infty}(-1)^n x^{2n}$, $-1<x<1$

(B) $\sum_{n=0}^{\infty}x^{2n}$, $-1<x<1$

(C) $\sum_{n=0}^{\infty}(-1)^n x^{2n}$, $-\infty<x<\infty$

(D) $\sum_{n=0}^{\infty}(-1)^n x^{n}$, $-1<x\leq 1$

89. The extreme value of $(x)^{1/x}$ is

(A) e

(B) $(\frac{1}{e})^e$

(C) $(e)^{1/e}$

(D) 1

90. If $y^2=P(x)$, a polynomial of degree 3, then $2\frac{d}{dx}(y^3\frac{d^2y}{dx^2})$ equals

(A) $P'''(x)+P'(x)$

(B) $P''(x)+P'''(x)$

(C) $P'''(x)P(x)$

(D) Not of these

91. The equation whose roots are the reciprocals of the roots of $x^3 + px^2 + r = 0$ is

(A) $x^3 + \frac{1}{p}x^2 + \frac{1}{r} = 0$

(B) $\frac{1}{r}x^3 + \frac{1}{p}x + 1 = 0$

(C) $rx^3 + px^2 + 1 = 0$

(D) $rx^3 + px + 1 = 0$

92. If the roots of the equation $x^n - 1 = 0$ are $1, \alpha_1, \alpha_2, \dots, \alpha_n$, then $(1-\alpha_1)(1-\alpha_2)\dots$ $(1-\alpha_n)$ is equal to

(A) 0

(B) 1

(C) n

(D) n+1

93. The solution of system of linear equations: $x + 2y + 3z = 0, 3x + 4y + 4z = 0,$ $7x + 10y + 12z = 0$ is

(A) x =0, y =0, z =0

(B) x =1, y =1, z =1

(C) x =1, y =2, z =3

(D) None of these

94. The eigen values of matrix $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ are

(A) 0, 0, 0

(B) 0, 0, 1

(C) 0, 0, 3

(D) 1, 1, 1

95. Two square matrices A and B are similar if

(A) A = B

(B) $B = P^{-1}AP$

(C) $A' = B'$

(D) $A^{-1} = B^{-1}$

96. If $f(x)$ is continuous in the closed interval [a, b], differentiable in (a, b) and $f(a) = f(b)$, then there exists at least one value c in (a, b) such that $f'(c)$ is equal to

(A) 1

(B) -1

(C) 2

(D) 0

97. If $A = f_{xx}(a,b), B = f_{xy}(a,b), C = f_{yy}(a,b)$, then $f(x,y)$ will have a maximum at (a, b) if

(A) $f_x = 0, f_y = 0, AC < B^2$ and $A < 0$

(B) $f_x = 0, f_y = 0, AC = B^2$ and $A > 0$

(C) $f_x = 0, f_y = 0, AC > B^2$ and $A > 0$

(D) $f_x = 0, f_y = 0, AC > B^2$ and $A < 0$

98. If $z = sin^{-1}\frac{\sqrt{x^2+y^2}}{x+y}$, then $x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y}$ is

(A) 0 (B) 1/2

(C) 1 (D) 2

99. Laplace transform of $t^4 e^{-at}$ is

(A) $\frac{4!}{(s+a)^4}$ (B) $\frac{4!}{(s+a)^5}$

(C) $\frac{4!}{(s-a)^4}$ (D) $\frac{5!}{(s-a)^5}$

100. $L^{-1}\frac{1}{s(s^2+1)}$ is

(A) $1 + \sin t$ (B) $1 - \sin t$

(C) $1 + \cos t$ (D) $1 - \cos t$

Section B (Short Answer Question)

Instruction: Attempt any 10 questions in all. All questions carry equal marks.

Each question is of 5 marks.

1. If $y = (sin^{-1}x)^2$, show that $(1 - x^2)y_{n+2} - (2n + 1)xy_{n+1} - n^2y_n = 0.$

2. In the Mean value theorem: $f(x + h) = f(x) + hf(x + \theta h)$, show that $\theta = 1/2$ for $f(x) = ax^2 + bx + c$ in $(0, 1)$.

3. If the system of linear equations

$$ax + by + cz = 0, bx + cy + az = 0, cx + ay + bz = 0$$

has non-trivial solution , show that $a + b + c = 0$ or $a = b = c$.

4. Reduce the matrix $A = \begin{bmatrix} -1 & 2 & -2 \\ 1 & 2 & 2 \\ -1 & -1 & 0 \end{bmatrix}$ to the diagonal form.

5. Solve the linear partial differential equation: $(x^2 - y^2 - z^2)p + 2xyq = 2xz$ where $p = \frac{\partial z}{\partial x}$ and $q = \frac{\partial z}{\partial y}$ are first order partial derivatives, respectively.

6. Find the curves on which the functional $\int_0^1 [(y')^2 + 12xy]\, dx$ with $y(0) = 0$ and $y(1) = 1$ can be extremised.

7. Discuss the convergence of the series: $1 - \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} - \frac{1}{\sqrt{4}} + \cdots$

8. Find the values of 'a' and 'b' such that $\lim_{x \to 0} \frac{x(a+bc \quad)-csinx}{x^5} = 1$.

9. If $\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$, $|\vec{r}| = r$ and $\hat{r} = \frac{\vec{r}}{r}$, then show that $grad\left(\frac{1}{r}\right) = -\frac{\hat{r}}{r^2}$.

10. Evaluate the line integral of $\vec{v} = x^2\hat{\imath} - 2y\hat{\jmath} + z^2\hat{k}$ over the straight line path from (-1, 2, 3) to (2, 3, 5).

11. A bag contains 4 red and 3 black balls. A second bag contains 2 red and 4 black balls. One bag is selected at random. From the selected bag one ball is drawn. Find the probability that the ball drawn is red.

12. The joint probability mass function of (x, y) is given by

$p(x,y) = K(3x + 5y), x = 1,2,3; y = 0,1,2$. Find the marginal distributions of X.

13. Using graphical method, solve the following linear programming problem

$$Maximize\ Z = 2x_1 + 3x_2$$

Subject to

$$x_1 - x_2 \leq 2, x_1 + x_2 \geq 4$$

$$x_1, x_2 \geq 0$$

14. Write the dual of the following linear programming problem

$$\text{Maximize } Z = 4x_1 + 9x_2 + 2x_3$$

Subject to $2x_1 + 3x_2 + 2x_3 \leq 7, 3x_1 - 2x_2 + 4x_3 = 5$

$$x_1, x_2, x_3 \geq 0$$

Section C (Long Answer Question)

Instruction: Attempt any two questions in all. All questions carry equal marks. Each question is of 25 marks.

1. Prove that

   (i) $\beta(m, 1/2) = 2^{2m-1}\beta(m, m)$

   (ii) $\Gamma(m)\Gamma(m + 1/2) = \frac{\sqrt{\pi}}{2^{2m-1}}\ \Gamma(2m)$, hence evaluate $\Gamma(\frac{1}{4})\Gamma(\frac{3}{4})$.

2. Find the Eigen values and Eigenvectors of the matrix A= $\begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$.

3. Evaluate $\int_S\ \vec{F}.\vec{N}ds$ where $\vec{F} = 2x^2y\hat{\imath} - y^2\hat{\jmath} + 4xz^2\hat{k}$ and S is the closed surface of the region in the first octant bounded by the cylinder $y^2 + z^2 = 9$ and the planes $x = 0, x = 2, y = 0$ and $z = 0$.

4. Solve the second order non-linear partial differential equation

$$(x - y)(xr - xs - ys + yt) = (x + y)(p - q)$$

where $= \frac{\partial^2 z}{\partial x^2}, t = \frac{\partial^2 z}{\partial y^2}$ and $s = \frac{\partial^2 z}{\partial x \partial y}$.